properties of reciprocal determinants the minors of the first row in (6) with their proper signs are equal to corresponding terms of the first row in (5) multiplied by D^2, *i. e.*

$$\left.\begin{array}{r}\begin{vmatrix} a & -b & c \\ -d & a & -b \\ c & -d & a \end{vmatrix} = xD^2 \\ \\ -\begin{vmatrix} -d & -b & c \\ c & a & -b \\ -b & -d & a \end{vmatrix} = yD^2 \end{array}\right\} \ldots (7),$$

etc. Solving (7) we have. after making a few changes of sign,

$$x = \frac{\begin{vmatrix} a & b & c \\ d & a & b \\ c & d & a \end{vmatrix}}{\begin{vmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{vmatrix}^{\frac{2}{3}} = \triangle}.$$

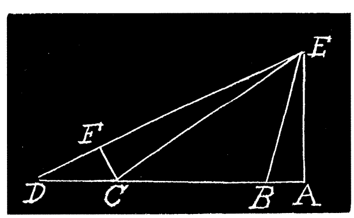
$$y = \frac{\begin{vmatrix} d & b & c \\ c & a & b \\ b & d & a \end{vmatrix}}{\triangle} \qquad z = \frac{\begin{vmatrix} d & a & c \\ c & d & b \\ b & c & a \end{vmatrix}}{\triangle} \qquad w = \frac{\begin{vmatrix} d & a & b \\ c & d & a \\ b & c & d \end{vmatrix}}{\triangle}$$

GEOMETRY.

372. Proposed by DANIEL KRETH, Oxford, Iowa.

In the right triangle ADE right angle A, are given: $AB=9$, $BC=280$, $CD=35$, angle AEB=angle CED; required the distance AE.

I. Solution by A. H. HOLMES, Brunswick, Me.



For $AB=9$, $BC=280$, and $CD=35$, the points B and C must be on the line AD. Otherwise the problem would be indeterminate. Let $AE=x$. Then $DE=\sqrt{(x^2+324^2)}$, $CE=\sqrt{(x^2+289^2)}$, and $BE=\sqrt{(x^2+9^2)}$.

From C let fall perpendicular CF to DE at F. To find EF we have

$$\sqrt{(x^2+324^2)} : \sqrt{(x^2+289^2)}+35 = \sqrt{(x^2+289^2)}-35 : \frac{x^2+289^2-35^2}{\sqrt{(x^2+324^2)}}.$$

$$\therefore EF=\frac{2x^2+324^2+389^2-35^2}{2\sqrt{(x^2+324^2)}}.$$

Then since $CED=AEB$ we have:

$$\sqrt{(x^2+289^2)}:\frac{2x^2+324^2+289^2-35^2}{2\sqrt{(x^2+324^2)}}=\sqrt{(x^2+9^2)}:x.$$

Squaring and reducing, $x^4-13259.64x^2=620781241.4125$.
$\therefore x=180.03+$

II. **Solution by B. KRAMER, E. M., and J. E. SANDERS, Weather Bureau, Columbus, Ohio.**

$\frac{\text{Area } AEB}{\text{Area } CED}=\frac{9}{35}$. Also the triangles having one angle equal,

$$\frac{x\sqrt{(x^2+9^2)}}{\sqrt{[(x^2+17^4)(x^2+18^4)]}}=\frac{9}{35}.$$

Squaring, $35^2x^2(x^2+9^2)=9^2(x^2+17^4)(x^2+18^4)$, or
$2^3.11.13x^4-17^2.2^2.9^4x^2-17^4.9^4.2^4.3^4=0$.

Solving for x^2, $x^2=\frac{17^2.9^4\pm\sqrt{(17^4.9^8+17^4.9^4.2.3^42^4.11.13)}}{2.11.13}$

$$=\frac{17^2.9^4\pm17^29^3\sqrt{(35^2)}}{2.11.13}=\frac{17^29^3(9\pm35)}{2.11.13}.$$

Taking the $+$ sign, the one real solution for x^2 is

$x^2=\frac{17^2.9^2.3^2.2^2.11}{2.11.13}=\frac{17^2.9^2.3^2.2}{13}$. $x=\pm17.9.3\sqrt{\frac{2}{13}}=\pm180.0346$. KRAMER.

An easier and more general result is obtained by using the tangent formula, thus:

$\tan AEB=9:x\ldots(1)$.

$$\tan DEC=\frac{\tan DEA-\tan CEA}{1+\tan DEA\times\tan CEA}=\frac{\frac{324-289}{x}}{1+\frac{324.289}{x^2}}\ldots(2).$$

Angles (1) and (2) being equal:

$$\frac{35x^2}{x(x^2+17^2.18^2)}=\frac{9}{x}, \text{ or } 26x^2=3^2.17^2.18^2;\ x^2=\frac{3^2.17^2.18^2}{26}.$$

$$x=\pm\frac{3.17.18}{\sqrt{26}}=\pm 3.17.9\sqrt{\tfrac{2}{13}}=\pm 180.0346.$$

SANDERS AND KRAMER.

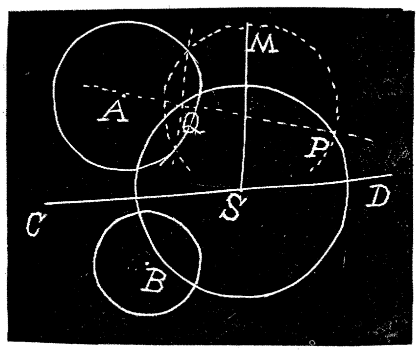
Also solved by V. M. Spunar and J, Scheffer.

373. Proposed by S. LEFSEHETZ, East Pittsburg, Pa.

Draw a circle passing through a given point and orthogonal to two given circles.

Solution by A. R. MAXSON, Columbia University, New York City.

Let P be the given point, and A and B the centers of the given circles. Draw CD, the radical axis of circles A and B. The required circle must have its center in CD. Again, remembering that the common chord of circle A and a circle described on AP as diameter cuts AP in a point Q, inverse of P with respect to circle A, construct Q and draw MS, the perpendicular bisector of QP cutting CD in S. S is the center of a circle of radius SP that passes through P and is orthogonal to circles A and B. We know that any circle through two points inverse with respect to a given circle is orthogonal to that circle. MS is, in our case, the locus of centers of circles through the inverse points P and Q. In particular, then, the circle of center S, S being in CD, must satisfy all the conditions of the problem.



Also solved by J. Scheffer, V. M. Spunar, S. G. Barton, C. N. Schmall, and the Proposer.

CALCULUS.

296. Proposed by C. N. SCHMALL, New York City.

Two currents C_1 and C_2 produce deflections ϕ_1, ϕ_2, respectively, in a tangent galvanometer. When is $(\phi_1-\phi_2)$ a maximum?

I. Solution by the late G. B. M. ZERR, Ph. D.

Let $G=2\pi n/r$=principal constant of the galvanometer. H=earth's horizontal magnetic force. $H/G=1/C$. Then $\tan\phi_1=CC_1$, $\tan\phi_2=CC_2$.

$\therefore u=\phi_1-\phi_2=\tan^{-1}(CC_1)-\tan^{-1}(CC_2)$.